Course Structure for Integrated MSc. in Mathematics

Course #	Credits		Course name
M 101 -	3	-	General Mathematics for all students
ML 101 -	2	-	Math Lab
M 201 -	3	-	General Mathematics for all students
ML 201 -	2	-	Math Lab
M 301 -	4	-	Real Analysis
M 302 -	4	-	Algebra - I
M 303 -	4	-	Complex Analysis
AM 301 -	2	-	Foundations of mathematics, logic and elementary number theory
ML 301 -	2	-	Math Lab
M 401 -	4	-	Probability and Statistics
M 402 -	4	-	Calculus of several variables
M 403 -	4	-	Algebra – II
AM 401 -	2	-	Discrete Mathematics and graph theory
ML 401 -	2	-	Math Lab
M 501 -	4	-	Topology
M 502 -	4	-	Differential Equations
M 503 -	4	-	Representation theory of finite groups
EM 501 -	4	-	Elective (Stream): Measure Theory
EM 502 -	4	-	Elective (Stream): Calculus on manifolds/Cryptology
PM 501 -	4	-	Project/Seminar
M 601 -	4	-	Functional Analysis
M 602 -	4	-	Differential Geometry
M 603 -	4	-	Optimization and calculus of variations
EM 601 -	4	-	Elective (Stream): Harmonic analysis
EM 602 -	4	-	Elective (stream):Commutative algebra
PM 601 -	4	-	Project/Seminar
M 701 -	4	-	Advanced PDE
M 702 -	4	-	Advanced Probability and Stochastic Process
EM 701 -	4	-	Elective(stream): Information and coding theory/Representation of linear lie groups
EM 702 -	4	-	Elective(stream):(Analytic/Algebraic) Number Theory
PM 701,702 -	4 X 2	-	Project/Seminar
M 801 -	4	-	Algebraic Topology
M 802 -	4	-	Nonlinear analysis
EM 801 -	4	-	Elective (stream): Algebraic graph theory
EM 802 -	4	-	Elective (stream):Theory of computation/Mathematical Logic
PM 801,802 -	4 X 2	-	Project/Seminar
EM 901 -	4	-	Elective (stream):
EM 902 -	4	-	Elective (stream):
DM 901-904 -	4 X 4	-	Dissertation
EM 1001 -	4	-	Elective (stream):
EM 1002-	4	-	Elective (stream):
DM 1001-1004 -	4 X 4	-	Dissertation

Compulsory Courses

1. Real Analysis
2. Algebra – I
3. Complex Analysis
4. Foundations of mathematics, logic and elementary number theory
5. Probability and Statistics
6. Calculus of several variables
7. Algebra – II
8. Discrete Mathematics and graph theory
9. Topology
10. Differential Equations
11. Representation theory of finite groups
12. Functional Analysis

13. Differential Geometry
14. Optimization and calculus of variations
15. Advanced PDE
16. Advanced Probability and Stochastic Process
17. Algebraic Topology
18. Nonlinear analysis

Elective Courses

1. Lie groups and Lie algebra
2. Analytic Number Theory
3. Algebraic Geometry
4. Algebraic Number Theory
5. Measure Theory
6. Calculus on Manifolds
7. Algebraic Graph Theory
8. Commutative algebra
9. Ergodic Theory
10. Harmonic analysis
11. Advanced Complex Analysis
12. Algorithm
13. Algebraic Computation
14. Finite Field and its Applications
15. Algebraic and Differential Topology
16. Advanced Functional Analysis
17. Operator Theory
18. Cryptology
19. Theory of Computation
20. Information and Coding Theory
21. Advanced Linear Algebra
22. Representations of Linear Lie Groups
23. Mathematical Logic
24. Special Topics *(to be suggested by the faculty)*

Syllabus

Compulsory Courses:

Semester 1

M 101: Linear Equations, matrices, Vector spaces, Groups,{15} Elementary differential equations (solution techniques for first and second order ordinary differential equations (ODE)){17}, partial differentiation, line and surface integrals, elementary vector calculus (gradient, divergence and curl).{10}

References:

1. Tom M. Apostol, Mathematical Analysis. Second Edition, Addison Wesley Publishing Company , 1974.
2. Tom M. Apostol, Calculus, Vol. 1: One-Variable Calculus with an Introduction to Linear Algebra, Wiley; 2 edition, 1967.
3. Tom M. Apostol, Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra with Applications, Wiley; 2 edition, 1969.
4. Tom M. Apostol, Linear Algebra: A First Course, with Applications to Differential Equations, Wiley-Interscience; 1 edition , 1997.
5. George F. Simmons and Steven G. Krantz, Differential Equations: Theory, Technique and Practice , McGraw Hill Higher Education, 2006.
6. George F. Simmons, Differential Equations with Applications and Historical Notes, 2nd edition McGraw-Hill Higher Education ,1991.

ML 101: Introduction to computer and algorithm, Languages:C/C++, Packages: Latex, MATLAB, MAPLE, MAXIMA, MATHEMATICA.

References:

1. Brian W. Kernighan, Dennis M. Ritchie: *The C Programming Language*. Prentice Hall.
2. E. Balaguruswamy: *Programming in ANSI C*.
3. B. Stroustrup: *The C++ Language*. Addison-Wesley.
4. E. Balaguruswamy: *Object-Oriented programming with C++*.
5. T. Cormen, C. Leiserson, R. Rivest, C. Stein: *Introduction to Algorithms*. McGraw-Hill Science.
6. Leslie Lamport: LaTex: A Document Preparation System. Addison-Wesley Professional 1994.
7. References for Matlab, Mathematica etc are as per instructor's reccomendation.

Semester 2

M 201: Real and complex numbers{8} Sequence, Series, Radius of convergence, {10} Differentiation, Integration, Fundamental theorem of calculus,{14} Taylor series, Binomial, Exponential, Logarithmic functions.{10}

References:

1. Tom M. Apostol, Mathematical Analysis. Second Edition, Addison Wesley Publishing Company , January 1974.
2. Tom M. Apostol, Calculus, Vol. 1: One-Variable Calculus with an Introduction to Linear Algebra, Wiley; 1967.
3. Robert G. Bartle and Donald R. Sherbert , Introduction to Real Analysis, 3rd Edition, Wiley; 1999.
4. Walter Rudin ,Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976 .

ML 201:

i. Number representation and errors in computation: Representation of integers and fractions, floating point arithmetics, source of errors, loss of significance, error propagation, computation of error estimation.
ii. Solution of non-linear equations: Bisection method, secant method, Newton's method, Muller's method.
iii. Interpolations:Polynomial interpolation, Newton divided difference, cubic spline interpolation.
iv. Approximation of functions: Weierstrass and Taylor expanasion, least square approximation.
v. Numerical integration and differentiation: Trapezoidal, Simpson's and Newton-Cotes rule, Guassian quadrature and orthogonal polynomial, numerical differentiation.
vi. Numerical solution of ODE: Euler's method, Runge-Kutta methods, multistep formula, Predictor-Corrector methods, Boundary value methods.

References:

1. Kendell A. Atkinson: *An introduction to numerical analysis.* John Wiley and Sons.
2. S.D. Conte, C. De Boor: *Elementary Numerical Analysis: An Algorithmic Approach*. Tata McGraw-Hill.
3. W.H. Press et. al.: *Numerical Recipes: The Art of Scientific Computing*. Cambridge University press.
4. Eugene Isaacson, Herbert Bishop Keller: *Analysis of Numerical Methods*. Dover Publications 1994.
5. A. Iserles: *A First Course in Numerical Analysis of Differential Equations*. Cambridge University Press.

Semester 3

M 301: Real Analysis

i. Metric topology (compactness, connectedness, continuity).{10}
ii. Riemann-Stieltjes integral, functions of bounded variation.{7}
iii. Sequence and series of functions, uniform convergence.{7}
iv. Arzela-Ascoli Theorem.{2}
v. Fourier series.{4}
vi. Lebesgue integral {12}

References:

1. George F. Simmons: *Introduction to Topology and Modern Analysis.* Krieger Pub Co,2003.
2. Walter Rudin ,*Principles of Mathematical Analysis*, Third Edition, McGraw-Hill; 1976.
3. Robert G. Bartle and Donald R. Sherbert , *Introduction to Real Analysis*, 3rd Edition, Wiley; 1999.
4. G. De Barra, *Measure Theory and Integration*, New Age International, 1981.
5. D. L. Cohn, *Measure Theory*, Springer, 1996.
6. H. L. Royden, *Real Analysis,* 3rd Edition, Pearson/Prentice Hall of India, 1988.

M 302: Algebra I

i. Groups, subgroups, homomorphisms {5}
ii. modular arithmetic {2}
iii. quotient groups, isomorphism theorems {3}
iv. groups acting on sets, permutation groups, matrix groups {10}
v. Sylow's theorems {12}
vi. Finite dimensional vector spaces, basis, eigenvalues and eigenvectors, canonical forms, diagonalization {10}

References:

1. Topics in Algebra – by – I. N. Herstein
2. Contemporary Abstract Algebra – by – J. A. Gallian
3. A First Course in Abstract Algebra – by – J. B. Fraleigh
4. Algebra – by – M. Artin
5. Linear algebra – by – K. M. Hoffman and R. Kunze
6. Introduction to Abstract Algebra – by – W. K. Nicholson
7. David S. Dummit, Richard M. Foote: Abstract Algebra. Wiley 2003.

M 303: Complex Analysis

i. The Cauchy-Riemann equations {2}
ii. Power series and holomorphy {8}
iii. Line integrals, the exponential map and the logarithm {3}
iv. Cauchy integral formula and its consequences {4}
v. Cauchy's theorem {2}
vi. Zeros, poles and singularities of holomorphic functions {6}

vii. The open mapping theorem and the argument principle {5}
viii. Maximum modulus principle, Schwarz lemma {4}
ix. Residues and the residue calculus {8}

References:

1. Theodore W. Gamelin, Complex Analysis, Springer; Corrected edition, 2003.
2. Elias M. Stein and Rami Shakarchi ,*Complex Analysis,* Princeton University Press, 2003.
3. R. V. Churchill and J. W. Brown, *Complex Variables and Applications*, 5th Edition, McGraw-Hill, 1990.
4. L. V. Ahlfors, Complex analysis. An introduction to the theory of analytic functions of one complex variable. McGraw-Hill Book Co., 1978.
5. W. Rudin, Real and complex analysis. McGraw-Hill Book Co., 1987.
6. J. B. Conway, *Functions of one complex variable*. II. Graduate Texts in Mathematics, 159.Springer-Verlag, 1995.
7. Reinhold Remmert: Theory of Complex Functions. Springer 1998.

ML 301:

i. Numerical linear algebra: Solution of linear equations by eliminations, Eigen value problem, canonical forms of matrices.
ii. Solution of partial differential equations: Finite difference methods.

References:

1. P. G. Ciarlet: *Introduction to Numerical Linear Algebra and Optimisation.Cambridge.* University Press 1989.
2. S.D. Conte, C. De Boor: *Elementary Numerical Analysis: An Algorithmic Approach*. Tata McGraw-Hill.
3. W.H. Press et. al.: *Numerical Recipes: The Art of Scientific Computing*. Cambridge University press.
4. Kendell A. Atkinson: *An introduction to numerical analysis.* John Wiley and Sons.
5. J.W. Thomas: *Numerical Partial Differential Equation*s. Springer 1998.
6. Lloyd N. Trefethen, David Bau: *Numerical Linear Algebra*. SIAM: Society for Industrial and Applied Mathematics 1997.
6. A. Iserles: *A First Course in Numerical Analysis of Differential Equations*. Cambridge University Press.

AM 301: Foundations of mathematics, logic and elementary number theory

i. Dedekind cuts {4}
ii. Ordinal and cardinal numbers, countable and uncountable sets {4}
iii. Propositional and quantified logic {4}
iv. Divisor function {2}
v. Euler Phi functions
vi. Fermat's little theorem, Wilson's theorem, Euler's theorem {4}
vii. Quadratic reciprocity law {2}
viii. Primitive roots {2}

References:

1. Walter Rudin: Principles of Mathematical Analysis. McGraw-Hill Science, 1976.
2. D. M. Burton: Elementary Number Theory. Mc Graw Hill, 2002.
3. J.A. Jones and J.M. Jones: Elementary Number Theory. Springer, 1998.
4. E. Mendelson: Introduction to Mathematical Logic. Chapman & Hall, 1997.
5. J. R. Shoenfield, *Mathematical logic*. Addison-Wesley Publishing Co., 1967.
6. J. Kelly: The essence of Logic. PHI, 2002.

<u>Semester 4</u>

M 401: Probability and statistics

i. Combinatorial probability and urn models {2}
ii. Conditional probability, independence {2}
iii. Discrete and continuous sample spaces {4}
iv. Random variables, Distributions and density functions, mean and measures {5}
v. Moment generating functions - probability laws (binomial, geometric, negative binomial, hypergeometric, Poisson, uniform, exponential, gamma) {6}
vi. Standard discrete distributions uniform, binomial, Poisson, geometric, hypergeometric {3}
vii. Independence of random variables, joint and conditional discrete distributions {3}
viii. Densities: normal, exponential, gamma, Chi-square, beta, Cauchy {3}
ix. Expectation and moments of continuous random variables {2}
x. Transformation of univariate random variables {2}
xi. Tchebychev's inequality and weak law of large numbers {2}
xii. Inferential statistics, estimation of parameters by method of moments and maximum likelihood. {4}
xiii. Confidence intervals, simple and complex hypotheses and list of hypotheses, least square estimation, test of hypothesis, analysis of variance. {4}

References:

1. Harold J. Larson: Introduction to Probability Theory and Statistical Inference. Wiley 1982.
2. V. K. Rohatgi: An Introduction to Probability Theory and Mathematical Statistics. John Wiley & Sons 1976.

3. John Freund: Introduction to Probability. Dover Publications.
4. Marylees Miller, John E. Freund, Irwin Miller: John E. Freund's Mathematical Statistics: With Applications. Prentice Hall, 2003.
5. William Feller: Introduction to Probability Theory and Its Application (Vol 1 and vol. 2). Wiley.
6. G. R. Grimmett, David R. Stirzaker: Probability and Random Processes. Oxford University Press, 2001.

M 402: Calculus of several variables

i. Differentiable maps from R^n - R^m {6}
ii. Df(w) as a linear map discussions {4}
iii. Higher derivatives, chain rule, Taylor expansion, local maxima and minima, Lagrange multiplier {8}
iv. Multiple integrals, existence of Riemann integrals for sufficiently well-behaved functions on a rectangle {6}
v. Change of variable formula with examples {3}
vi. Inverse and Implicit function theorems (no proofs) {2}
vii. Line and surface integrals {3}
viii. Divergence, gradient, curl {5}
ix. Green's, Stokes & Gauss theorems {5}

References:

1. Wendell Fleming: Functions of Several Variables. Springer 1987.
2. Tom M. Apostol, Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra with Applications, Wiley; 2 edition , June 1969.
3. Michael Spivak, Calculus On Manifolds: A Modern Approach To Classical Theorems Of Advanced Calculus, Westview Press, 1971
4. Walter Rudin ,Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976 .

M 403: Algebra – II

i. Ideals, Factorization in a ring, Euclidean domain, principal ideal domain, unique factorization domain {10}
ii. Field of fractions, Gauss lemma {6}
iii. Fields, field extension, Galois theory {8}
iv. Finitely generated modules over a PID and their representation {6}
v. Structure theorem for finitely generated Abelian groups {6}
vi. Rational form and Jordan form of a matrix {6}

References:

1. Topics in Algebra – by – I. N. Herstein
2. Contemporary Abstract Algebra – by – J. A. Gallian
3. A First Course in Abstract Algebra – by – J. B. Fraleigh
4. Algebra – by – M. Artin
5. Linear algebra – by – K. M. Hoffman and R. Kunze
6. Introduction to Abstract Algebra – by – W. K. Nicholson
7. David S. Dummit, Richard M. Foote: Abstract Algebra. Wiley 2003.

ML 401: Solving System of linear equations by conjugate gradient methods, various methods of optimization; sorting and searching algorithms; computation of special functions.

References:

1. P. G. Ciarlet: *Introduction to Numerical Linear Algebra and Optimisation.Cambridge.* University Press 1989.
2. Kendell A. Atkinson: *An introduction to numerical analysis*. John Wiley and Sons.
3. W.H. Press et. al.: *Numerical Recipes: The Art of Scientific Computing*. Cambridge University press.
4. T. Cormen, C. Leiserson, R. Rivest, C. Stein: *Introduction to Algorithms*. McGraw-Hill Science.
5. Stephen Boyd, Lieven Vandenberghe: *Convex Optimization*. Cambridge University Press 2004.

AM 401: Discrete Mathematics and Graph theory

i. Counting principles, recursion, generating functions, mathematical induction {10}
ii. Graphs, sub-graphs, isomorphism, planar graphs, trees, graph coloring {12}

References:

1. F. Roberts and B. Tesman: Applied Combinatorics. Pearson Education, 2005.
2. R. A. Brualdi: Introductory Combinatorics. Pearson Education, 2008.
3. G.E. Martin: Counting: The Art of Enumerative Combinatorics. Springer, 2001.
4. F. Harary: Graph Theory. Narosa Publishing House, 2001.
5. G.A. Bondy and U.S.R. Murty: Graph Theory. Springer, 2008.
6. Bela Bollobas:Extremal Graph Theory. Dover Publications.

<u>Semester 5</u>

M 501: Topology

i. Topological spaces, quotient spaces {12}
ii. Separation axioms (Urysohn's lemma, Tietze's extension theorem) {6}
iii. Filters and nets {4}
iv. connectedness and compactness {6}
v. Covering spaces, fundamental groups {14}

References:

1. T James Munkres , *Topology,* Second Edition, Prentice Hall, 2000.
2. K J¨anich. *Topology*, Springer, 1984.
3. M A Armstrong. *Basic Topology*. Springer, 1983.
4. G E Bredon, *Topology and Geometry,* Springer GTM 139, 1995.
5. W. S. Massey, *A basic course in algebraic topology*. Graduate Texts in Mathematics, 127.Springer-Verlag, 1991.

M 502: Differential Equations

i. Second order linear equations with constant coefficients {2}
ii. System of first order differential equations, equations with regular singular point {8}
iii. power series methods special ordinary differential equations from physics, some special functions like Bessel's function, Legendre polynomial, gamma function {8}
iv. Picard's theorem on existence and uniqueness of solution to first order ordinary differential equation {3}
v. Oscillations - Sturm Liouville theory {4}
vi. First order partial differential equations {5}
vii. The Laplace, Heat and the Wave equations {10}

References:

1. S. L. Ross, *Differential Equations*, 3rd Edition, Wiley, 1984.
2. E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall, 1995.
3. Earl A. Coddington and Norman Levinson ,*Theory of Ordinary Differential Equations* , Krieger Pub Co, 1984
4. Lawrence C. Evans, *Partial Differential Equations,* American Mathematical Society , 1998.
5. Fritz John: Partial Differential Equations. Springer 1995.

M 503: Representation theory of finite groups

i. Multilinear forms, tensor products, wedge product, Grassmann ring, symmetric product {18}
ii. complete reducibility, Schur's lemma, characters {6}
iii. Projection formulae, inducted representation {4}
iv. Frobenius reciprocity {4}
v. Representation of permutation groups
vi. Young's tableaux, Machke's theorem, sums, products, exterior and symmetric power of representations, applications to group rings, characters.

References:

1. J. P. Serre: Linear Representations of Finite Groups. Springer 1996.
2. William Fulton, Joe Harris: Representation Theory: A First Course. Graduate Texts in Mathematics, springer.

<u>Semester 6</u>

M 601: Functional Analysis

i. Normed linear spaces and continuous linear transformations {12}
ii. Hahn-Banach theorem (analytic and geometric versions) {4}
iii. Baire's theorem and its consequence - three basic principles of functional analysis (open mapping theorem, closed graph theorem and uniform boundedness principle) {12}
iv. Hilbert spaces - Riesz representation theorem, adjoint operator, etc. {4}
v. Compact operators {4}
vi. Spectral theorem for self adjoint compact operators {6}

References:

1. W. Rudin, *Functional analysis*. McGraw-Hill, Inc., 1991.
2. J. B. Conway, *A course in functional analysis*. Graduate Texts in Mathematics, 96. Springer-Verlag, 1990.
3. K. Yosida, *Functional analysis*. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.

M 602: Differential Geometry

i. Curvature and torsion for space curves {12}
ii. Surfaces in R^3 as 2-dimensional manifolds {4}
iii. Tangent spaces and derivatives of maps between manifolds {4}
iv. First & Second fundamental forms and the Gauss map {4}
v. Differential forms {4}
vi. Integration on surfaces {8}
vii. Gauss-Bonnet theorem {6}

References:

1. M. P. Do Carmo: Differential Geometry of Curves and Surfaces. Prentice Hall 1976.
2. M. P. Do Carmo: Differential Forms and Applications. Springer 2000.
3. John A. Thorpe: Elementary Topics in Differential Geometry. Springer 1994.

M 603: Optimization and calculus of variations

i. Linear programming, simplex method, duality {8}
ii. Applications - transportation problems {4}

iii. Calculus in normed linear spaces {5}
iv. Quadratic programming {4}
v. Conjugate gradients, etc.{4}
vi. Farkas - Minkowski lemma, Kuhn - Tucker condition {3}
vii. Calculus of variations: Euler-Lagrange equation and some sufficient conditions {8}
viii. Isoperimetric inequality {6}

References:

1. Frank H. Clarke: Optimization and Nonsmooth Analysis. Society for Industrial Mathematics 1987.
2. David G. Luenberger: Introduction to linear and non-linear programming.
3. Mokhtar S. Bazaraa, Hanif D. Sherali, C. M. Shetty : Nonlinear Programming: Theory and Algorithms. Wiley-Interscience 2006.
4. Bruce van Brunt: The Calculus of variations. Springer 2003.

<u>Semester 7</u>

M 701 : Advanced PDE

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewy's example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems—interior regularity, local existence.

References:

1. G. B. Folland, *Introduction to partial differential equations*. Princeton University Press,1995.
2. J. Rauch, *Partial differential equations*. Graduate Texts in Mathematics, 128. Springer-Verlag, 1991.
3. E. DiBenedetto, *Partial differential equations*. Birkhäuser Boston, Inc., 1995.
4. L. C. Evans, *Partial differential equations*. Graduate Studies in Mathematics, 19. American Mathematical Society, 1998.
5. Michael Renardy, Robert C. Rogers: *Introduction to Partial Differential Equations*. Springer 2004.

M 702 : Advanced Probability and Stochastic Process

Discrete-time Discrete-state Markov Chains, Classification of States, Recurrence, Transience, Stationary Distribution and Stability, Ergodicity, Reversibility.

Topics From :

a) Rates of convergence to stationarity, Dirichlet Form and Spectral gap methods,
b) Some Coupling methods with applications,
c) Random walk on Finite Groups,
d) Poisson Processes,
e) Continuous time Markov Chains , Birth-and-death processes,
f) Stationary processes,
g) Markov processes and generators,
h) Weak Convergence of probability measures on polish spaces including C[0, 1],
i) Brownian motion; construction, simple properties of paths,
j) Poisson processes,
k) Connections between Brownian Motion / Diffusion and PDE's.

References:

1. S. M. Ross, *Stochastic processes*. John Wiley & Sons, Inc., 1996.
2. R. N. Bhattacharya and E. C. Waymire, *Stochastic processes with applications*. A Wiley-Interscience Publication. John Wiley & Sons, Inc., 1990.
3. E. Giné, G. R. Grimmett and L. Saloff-Coste, *Lectures on probability theory and statistics*. Lecture Notes in Mathematics, 1665. Springer-Verlag, 1997.
4. P. Billingsley, *Convergence of probability measures*. John Wiley & Sons, Inc., 1999.
5. K. Ito, *Stochastic processes*. Lecture Notes Series, No. 16 Matematisk Institut, Aarhus Universitet, Aarhus 1969.
6. D. Revuz and M. Yor, *Continuous martingales and Brownian motion*. Grundlehren der Mathematischen Wissenschaften, 293. Springer-Verlag, 1999
7. Geoffrey R. Grimmett, David R. Stirzaker: Probability and Random Processes. Oxford University Press, 2001.

<u>Semester 8</u>

M 801 : Algebraic Topology

i. Singular homology functors and its axiomatic properties. Relations between fundamental group and first homology. Mayer-Vietoris sequence, computation of homology of spheres.Degree of maps with applications to spheres.
ii. Simplicial CW-complexes, cellular description of homology, comparison with singular theory.Computation of homology of projective spaces.

iii. Definition of singular cohomology, its fundamental properties, statement of universal coefficient theorem, Betti number and Euler characteristic, cup product, Poincare duality.

References:

1. M. J. Greenberg, *Lectures on algebraic topology*. W. A. Benjamin, Inc., 1967.
2. J. R. Munkres, *Elements of algebraic topology*. Addison-Wesley Publishing Company, 1984.
3. R. Bott and L. W. Tu, *Differential forms in algebraic topology*. Graduate Texts in Mathematics, 82. Springer-Verlag, 1982.
4. Allen Hatcher, *Algebraic Topology,* Cambridge University Press; 2001.

M 802 : Nonlinear analysis

Calculus in Banach spaces, inverse and multiplicit function theorem, fixed point theorems of Brouwer, Schauder and Tychonoff, fixed point theorems for nonexpansive and set-valued maps, predegree results, compact vector fields, homotopy, homotopy extension, invariance theorems and applications.

References:

1. S. Kesavan, Nonlinear Functional analysis. Hindustan Book Agency, 2004.

***() Refers to number of lectures**

<u>Elective Courses:</u>

1. Lie groups and Lie algebra:

i. Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras e.g. Complex groups: GL(n, C), SL(n, C), SO(n, C), Groups of real matrices in those complex groups: GL(n, R), SL(n, R), SO(n, R), Isometry groups of Hermitian forms SO(m, n), U(m, n), SU(m, n). Finite dimensional representations of su(2) and SU(2) and their connection. Exhaustion using the lie algebra su(2).

ii. Lie algebras in general, Nilpotent, solvable, semisimple Lie algebra, ideals, Killing form, Lie's and Engel's theorem. Universal enveloping algebra and Poincare-Birkhoff-Witt Theorem (without proof).

iii. Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/ reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition g=k+p, examples of k and p for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartan's criterion for semisimplicity. Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition (statement only).

iv. If time permits, one of the following topics :
   a) A brief introduction to Harmonic Analysis on SL (2, R).
   b) Representations of Compact Lie Groups and Weyl Character Formula.
   c) Representations of Nilpotent Lie Groups

References:

1. J. E. Humphreys, *Introduction to Lie algebras and representation theory*. Graduate Texts in Mathematics, 9. Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari, *A first course on representation theory and linear Lie groups*. University Press, 2000.
3. Serge Lang, *SL(2, R)*. Graduate Texts in Mathematics, 105. Springer-Verlag, 1985.
4. W. Knapp, *Representation theory of semisimple groups. An overview based on examples*. Princeton Mathematical Series, 36. Princeton University Press, 1986.
5. Brian C. Hall: Lie Groups, Lie Algebras, and Representations: An Elementary Introduction. Springer 2004.

2. Analytic Number Theory:

Averages of mathematical functions, distribution of primes, weyl's, Kronecker's and Minkowski's theorems, characters, Reimann Zeta function and Dirichlet L-functions, Dirichlet's Theorem on primes in arithmetic progression, Functional equation and Euler product for L-functions, analytic proof of the prime number theorem. Hesse-Minkowski theorem.

References:

1. J. P. Serre: A course in Arithmetic. Springer-verlag, 1973.
2. H. Hesse: Number Theory. Springer-Verlag, 1980.

3. Algebraic Geometry:

Polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert Nullstellensatz, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Abelian differential, Riemann–Roch for curves.

References:

1. C. Musili, *Algebraic geometry for beginners*. Texts and Readings in Mathematics, 20. Hindustan Book Agency, 2001.

2. W. Fulton, *Algebraic curves. An introduction to algebraic geometry*. Advanced Book Classics. Addison-Wesley Publishing Company, Advanced Book Program, 1989.
3. K. Kendig, *Elementary algebraic geometry*. Graduate Texts in Mathematics, No. 44. Springer-Verlag, 1977.
4. R. Shafarevich, *Basic algebraic geometry. 1. Varieties in projective space*. Springer-Verlag,1994.
5. J. Harris, *Algebraic geometry. A first course*. Graduate Texts in Mathematics, 133. Springer-Verlag, 1995.
6. M. Reid, *Undergraduate algebraic geometry*. London Mathematical Society Student Texts, 12. Cambridge University Press, 1988.

4. Algebraic Number Theory:

Norms, traces and discriminants, Algebraic number fields, Dedekind domain, Unique factorization, Quadratic fields , biquadratic fields, The ideal class group, Cyclotomic extensions, The Dirichlet unit theorem, Local field.

References:

1. S. Alaca, K. S. Williams: Introductory Algebraic Number Theory. Cambridge.
2. Serge Lang: Algebraic Number Theory. GTM, Springer.
3. A. Frohlich, M. J. Taylor: Algebraic Number Theory. Cambridge studies in advance Mathematics 27.
4. H. Hesse: Number Theory. Springer-Verlag, 1980.

5. Measure Theory:

σ−algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions, *Lp*-spaces, Riesz representation theorem for the space C[0, 1].

References:

1. G. De Barra, Measure theory and integration.
2. J. Nevue, *Mathematical foundations of the calculus of probability*. Holden-Day, Inc., 1965.
3. I. K. Rana, *An introduction to measure and integration*. Narosa Publishing House, 1997.
4. P. Billingsley, *Probability and measure*. John Wiley & Sons, Inc., 1995.
5. W. Rudin, *Real and complex analysis*. McGraw-Hill Book Co., 1987.
6. K. R. Parthasarathy, *Introduction to probability and measure*. The Macmillan Co. of India, Ltd., 1977.

6. Calculus on Manifolds:

Differentiable manifolds, tangent bundle, vector bundles, vector fields, flows and the fundamental theorem of ODE's. Immersion, submersion, submanifolds and transversality, Differential forms and integration, Riemannian metrics. Riemannian connection on Riemannian manifolds, Gauss-Bonnet Theorem. Parallel transport, geodesics and geodesic completeness, the theorem of Hopf-Rinow.

References:

1. F. W. Warner, *Foundations of differentiable manifolds and Lie groups*. Graduate Texts in Mathematics, 94. Springer-Verlag, 1983.
2. S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*. Graduate Studies in Mathematics, 34. American Mathematical Society, 2001.
3. Glen E. Bredon: Topology and Geometry. Springer 1997.
4. Victor Guillemin, Alan Pollack: Differential Topology. Prentice Hall 1974.
5. John A. Thorpe: Elementary Topics in Differential Geometry. Springer 1994.

7. Algebraic Graph Theory:

Introduction,Spectrum of a graph,Complexity and determinant expansions,Colouring and the spectrum,The Laplacian of a graph,,General properties of graph automorphisms, Transitive and arc-tranisitve graphs, The spectrum and the group of automorphism

References:

1. N. Biggs: Algebraic Graph Theory . Cambridge Mathematical Library.
2. C. Godsil, G. Royle: Algebraic Graph Theory. Springer International Edition.

8. Commutative algebra:

Commutative rings, ideals, prime and maximal ideals, Noetherian Artinian ring,Prime decomposition and associate primes, Integral extensions, Valuation rings, Completion, Dimension theory, Exact sequences, Completions, Dimension theory, Cohen-Macaulay rings.

References:

1. M. F. Atiyah and I. G. Macdonald: Introduction to Commutative Algebra.
2. R. Y. Sharp: Steps in Commutative Algebra. LMS.

9. Ergodic Theory:

i. Measure preserving systems; examples: Hamiltonian dynamics and Liouville's theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product.
ii. Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlin's lemma. Recurrence in Topological Dynamics, Birkhoff's Recurrence theorem.

iii. Ergodicity, Weak-mixing and strong-mixing and their characterizations.

iv. Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyl's theorem.

v. The Isomorphism problem; conjugacy, spectral equivalence.

vi. Transformations with discrete spectrum, Halmos–von Neumann theorem.

vii. Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-Millan–Breiman Theorem.

viii. Flows. Birkhoff's ergodic Theorem and Wiener's ergodic theorem for flows. Flows built under a function.

References:

1. Peter Walters, *An introduction to ergodic theory*. Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
2. Patrick Billingsley, *Ergodic theory and information*. Robert E. Krieger Publishing Co., 1978.
3. M. G. Nadkarni, *Basic ergodic theory*. Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
4. H. Furstenberg, *Recurrence in ergodic theory and combinatorial number theory*. Princeton University Press, 1981.
5. K. Petersen, *Ergodic theory*. Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

10. Harmonic analysis:

Fourier transforms,the Schwartz space, Distribution and tempered distribution, Fourier Inversion and Plancherel theorem. Fourier analysis on L^p spaces. Maximal functions and bounded ness of Hilbert transform. Paley-Wiener Theorem for distribution. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem.

References:

1. Y. Katznelson, *An introduction to harmonic analysis*. Dover Publications, Inc., New York,1976.
2. E. Hernández and G. Weiss, *A first course on wavelets*. Studies in Advanced Mathematics.CRC Press, 1996.
3. Elias M. Stein and Guido Weiss: Introduction to Fourier Analysis on Euclidean Spaces (PMS-32).

11. Advanced Complex Analysis:

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities andcalculus of residues. Normal families, Arzela's theorem. Product developments, functions with prescribed zeroes and poles, Hadamard's theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations. Introduction to functions of several complex variables (if time permits)

References:

1. L. V. Ahlfors, *Complex analysis. An introduction to the theory of analytic functions of on complex variable*. McGraw-Hill Book Co., 1978.
2. J. B. Conway, *Functions of one complex variable*. II. Graduate Texts in Mathematics, 159.Springer-Verlag, 1995.
3. W. Rudin, *Real and complex analysis*. McGraw-Hill Book Co., 1987.
4. Reinhold Remmert: Theory of Complex Functions. Springer 1998.

12. Algorithm:

i. *Preliminaries*: Introduction to algorithms, growth of functions, time and space complexity measures, overview on data structures and algorithm design principles.

ii. *Sorting and searching*: Searching maximum, minimum, kth largest element in a [ordered-]list, binary search, buble sort, divide and conquer, heap sort, quick sort, merge sort, radix sort, their time complexity.

iii. *Dynamic Programming*: Matrix-chain multiplication, elements of dynamic programming, longest common subsequence.

iv. *Graph algorithms*: Graph searching- BFS, DFS, sortest first search, topological sort; connected and bi-connected components; spanning tree- the algorithms of Kruskal and Prim.

v. *Algebraic algorithms*: Winograd's and Strassen's matrix multiplication algorithm, evaluation of polynomials, DFT, FFT, efficient FFT implementation.

vi. *String processing*: String searching and pattern matching, Knuth-Morris -Pratt algorithm and analysis.

vii. *Computational geometry*: Line-segment properties, intersection of any pair of segments, finding the convex hull, finding the closest pair of points.

viii. *NP-completeness*: deterministic and non-deterministic algorithm, P and NP class, some NP-complete problems.

References:

1. Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman: Design and Analysis of Computer Algorithms. Addison-Wesley, 1974.
2. Thomas Cormen, Charles Leiserson, Ronald Rivest: Introduction to Algorithms. PHI, 1998.
3. E. Horowitz, S. Sahni: Fundamental of Computer Algorithms. Galgotia publication, 1987.

4. D.E. Knuth: The art of Computer Programming, Vol 1, vol. 2, vol 3. Addison-Wesley,

13. **Algebraic Computation:**
    i. *Algorithms on polynomials:* GCD, Barlekamp-Massey algorithm, factorization of polynomials over finite field, lattice reduction, factorization of polynomials over Z and Q.
    ii. *Matrix Computation:* Asymptotically fast matrix multiplication algorithms, symbolic and exact solutions of linear systems, Diaphontine analysis, normal forms over fields.
    iii. *Solving Systems of non-linear equations*: Groebner basis, Buchberger's algorithms, Complexity of Groebner basis computation, F4, F5.
    iv. *Algorithms for algebraic number theory*: Representation and operations on algebraic numbers, trace, norm, charecteristic polynomial, discriminant, integral bases, polynomial reduction, computing maximal order, algorithms for quadratic fields.
    v. *Elliptic curves:* Implementation of elliptic curve, algorithms for elliptic curves.

References:

1. Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman: Design and Analysis of Computer Algorithms. Addison-Wesley, 1974.
2. H. Cohen: A course in Computational Number Theory. Springer-verlag, 1993.
3. D.A. Cox, J.B. Little, D. O'shea: I deals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra, Springer-verlag, 1996.

14. **Finite Field and its Applications:**
    i. *Structure of finite fields:* Characterization, traces, norms and bases, Roots of irrudicible polynomial, unity and cyclotomic polynomial, representation of elements of finite field
    ii. *Polynomials over finite field:* Order of polynomials and primitive polynomial, construction of irruducible polynomial, binomials and trinomials, factorization of polynomials over small and large fields, calculation of roots of polynomials.
    iii. *Linear recurring sequances*: LFSR, characteristic polynomial, minimal polynomial, Berlekamp-Messey algorithm.
    iv. *Applications of finite field*: Applications in cryptography, coding theory, finite geometry, combinatorics.

References:

1. R. Lidl, H. Neiderreiter: Finite fields. Cambridge university press, 2000.

15. **Algebraic and Differential Topology:**

Alexander-Lefschetz duality in topological manifolds. De Rham cohomology of manifolds, deRham theorem, Stokes theorem. Computation of Cohomology rings of projective spaces, Borsuk-Ulam theorem. Higher homotopy groups, fibration, homotopy exact sequence of a pair andof a fibration. Poincare-Hopf theorem.

References:

1. R. Bott and L. W. Tu, *Differential forms in algebraic topology*. Graduate Texts in Mathematics,82. Springer-Verlag, 1982.
2. M. J. Greenberg, *Lectures on algebraic topology*. W. A. Benjamin, Inc., 1967
3. F. W. Warner, *Foundations of differentiable manifolds and Lie groups*. Graduate Texts in Mathematics, 94. Springer-Verlag, 1983.
4. Victor Guillemin, Alan Pollack: Differential Topology. Prentice Hall 1974.

16. **Advanced Functional Analysis :**

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounov’s theorem). Extreme points and Krein-Milman theorem. In addition, one of the following topics:

a) Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein-Smulian Theorem. Schauder Basis.
b) Banach algebras, spectral radius, maximal ideal space, Gelfand transform.
c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

References:

1. N. Dunford and J. T. Schwartz, *Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space*. Interscience Publishers John Wiley & Sons 1963.
2. Walter Rudin, *Functional analysis*. Second edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., 1991.
3. K. Yosida, *Functional analysis*. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.
4. J. Diestel and J. J. Uhl, Jr., *Vector measures*. Mathematical Surveys, No. 15. American Mathematical Society, 1977.

17. **Operator Theory:**
    i. Compact operators on Hilbert Spaces. a) Fredholm Theory b) Index

ii. C*-algebras—noncommutative states and representations, Gelfand-Neumark representation theorem.
iii. Von-Neumann Algebras; Projections, Double Commutant theorem, L^infinity functionalCalculus.
iv. Toeplitz operators

References:

1. W. Arveson, *An invitation to C*-algebras*. Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, *Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space*. Interscience Publishers John Wiley & Sons 1963.
3. R. V. Kadison and J. R. Ringrose, *Fundamentals of the theory of operator algebras. Vol. I.Elementary theory*. Pure and Applied Mathematics, 100. Academic Press, Inc., 1983.
4. V. S. Sunder, *An invitation to von Neumann algebras*. Universitext. Springer-Verlag, 1987.

18. Cryptology:

i. *Overview of Cryptology:* Cryptography and cryptanalysis, some simple cryptosystems (e.g., shift, substitution, affine, knapsack) and their cryptanalysis,classification of cryptosystems, concept of stream and block ciphers, , classification of attacks.
ii. *Information Theoretic Ideas:* Perfect secrecy, Shanon's theory, entropy, unicity distance.
iii. *Stream cipher*: LSFR based stream cipher, nonlinear and filter combiner model of stream cipher, RC4, Attacks on stream cipher (e.g., correlation attack, algebraic attacks etc).
iv. *Block cipher*: DES, linear and differential cryptanalysis, AES.
v. *Public-key cryprosystem*: RSA with implemetation, primility testing, integer factorization, El Gamal public-key cryptosystem, Discrete logarithm problem, elliptic curve cryptography.
vi. *Digital signature and hash functions:* Hash functions, security of hash functions, construction of hash functions, message authentication code, security for signature scheme, El Gamal signature scheme.
vii. *Secret sharing :* Shamir's threshold scheme, general access structure and secret sharing.

References:

1. A. J. Menezes, S. A. Vanstone, and P. C. Van Oorschot. Handbook of Applied Cryptography. CRC Pr Llc, 1996.
2. D. R. Stinson. Cryptography: Theory And Practice. CRC Pr Llc, 2006.
3. W. Stallings. Cryptography and network security. Prentice Hall, 2005.

19. Theory of Computation:

i. *Automata and Languages:* Finite automata, regular expression, [non-]regular languages, deterministic and non-deterministic automata, minimization of finite automata, pumping lemma and its applications. Context free grammaers, [non-]context free languages, Chomsky normal form, push down automata, pumping lemma for CFL.
ii. *Computibility*: Computable functions, primitive and recursive functions, universality, halting problem, recursive and recursively enumerable sets, diagonalization, reducibilty, Rice's theorem and its application, Turing machine and its variants, Churh-Turing thesis.
iii. *Complexity*: Time complexity of Turing machines, Classes P and NP, NP completeness, other time classes, the time hierarchy.

References:

1. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Automata Theory, Languages, and Computation. Addison Wesley 2006.
2. Harry Lewis, Christos H. Papadimitriou: Elements of the Theory of Computation. Prentice Hall 1997.
3. Michael Sipser: Introduction to the theory of computation. PWS Publishing 1997.

20. Information and Coding Theory:

i. *Information Theory:* Entropy, Huffman coding, Shannon-Fano coding, entropy of Markov process, channel and mutual information, channel capacity.
ii. *Error correcting codes:* Maximum likelihood decoding, nearest neighbour e decoding, linear codes, generator matrix and parity-check matrix, Hamming bound, Gilbert–Varshamov bound, binary Hamming codes, Plotkin bound, nonlinear codes, Reed–Muller codes, Cyclic codes, BCH codes, Reed–Solomon codes

References:

1. San Ling, Chaoping Xing: Coding Theory: A First Course. Cambridge Univ Pr, 2004.
2. Richard W. Hamming: Coding and Information Theory. Prentice Hall, 1986.
3. Vera Pless: Introduction to the Theory of Error-Correcting Codes. Wiley-Interscience, 1998.
4. Neil J. A. Sloane, Florence Jessie MacWilliams: Theory of Error Correcting Codes, Vol I and II. North-Holland, 1983.
5. S. Lin: An Introduction to Error-Correcting Codes. Prentice Hall, 1970.

21. Advanced Linear Algebra*:*

Review of basic Linear Algebra, Canonical factorization, Q- forms, Bilinear forms, Hermitian forms, Duality, Tensor product, Courant- Fishcher minimax and related theorems, Perron-Frobenius theory, Matrix Norm, Matrix stability and

Inequality, Generalized inverse.

References:

1. Advanced Linear Algebra (GTM) – by – Steven Roman
2. Matrix Analysis – by- R. A. Horn and C. R. Johnson
3. Matrix Analysis – by – R. Bhatia
4. Linear Algebra – by – K. Hoffman and R. Kunze

22. Representations of Linear Lie Groups:

Introduction to topological group, Haar measure on locally compact group, Representation theory of compact groups, Peter Weyl theorem, Linear Lie groups, Exponential map, Lie algebra, Invariant Differentail operators, Representation of the group and its Lie algebra. Fourier analysis on $SU(2)$ and $SU(3)$. Representation theory of Heisenberg group . Representation of Euclidean motion group.

References:

1. J. E. Humphreys, *Introduction to Lie algebras and representation theory*. Graduate Texts in Mathematics, 9. Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari, *A first course on representation theory and linear Lie groups*. University Press, 2000.
3. Mitsou Sugiura, Unitary Representations and Harmonic Analysis.
4. Sundaram Thangavelu, Harmonic Analysis on the Heisenberg Group, Progress in Mathematics.
5. Sundaram Thangavelu : An Introduction to the Uncertainty Principle: Hardy's Theorem on Lie Groups by , Progress in Mathematics.

23. Mathematical Logic:

Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem. First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g. , theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godel’s first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

References:

1. J. R. Shoenfield, *Mathematical logic*. Addison-Wesley Publishing Co., 1967.
2. E. Mendelson: Introduction to Mathematical Logic. Chapman & Hall, 1997.

24. Special Topics *(to be suggested by the faculty):*